Course title: Real Analysis and Optimisation			
Course code: MPE 117	No. of credits: 4	L-T-P: 48-12-0	Learning hours: 60
Pre-requisite course code and title:			
Department: Department of Policy and Management Studies			
Course coordinator: Dr. Shantanu De Roy	Course instructor: Guest Faculty		
Contact details:			
Course type: Core	Course offered in: 1st semester		
Course description: This is an introductory course on the fundamentals of mathematics that is heavily used in certain disciplines of economics – microeconomics, macroeconomics, etc. The course is broadly divided into two parts: one is real analysis and the other is optimisation. The course covers sequences, limits, continuity, differentiability, convex analysis, unconstrained optimisation, optimisation with equality and inequality constraints.			
Course objective: 1. To develop the core concepts of mathematics that are used in economics. 2. To develop mathematical sophistication and proof writing. 3. To build analytical and technical skills by rigorous analysis. 4. To apply these techniques and skills in economic applications.			

Course contents

S.No	Topics	L	T	P
1	**Preliminaries.** Logic: Statements, conjuction and disjunction, implications, necessity and sufficiency, contrapositive and converse, quantifiers, theorems and proofs. Sets and functions: domain, range, types of functions, direct and inverse images, inverse and composite functions. Cardinality: finite sets, denumerable sets, countability.	4		
2	**Real Analysis I.** Supremum and infimum: upper bound, lower bound, bounded sets, completeness property, Archimedean property. Sequences and limits: metric spaces, sequences on the real line, convergence, limit point, cluster point, divergence, bounded sequences, limit supremum, limit infimum, algebraic limit theorem, squeeze theorem, monotone sequences, monotone convergence theorem, subsequence, Bolzano-Weierstrass theorem, Cauchy criteria, completeness, functional limits.	9	3	
3	**Real Analysis II.** Topology: open ball, neighborhood, interior points, closure points, accumulation points, interior of a set, closure of a set, open sets, closed sets, characterisations, open cover, compact sets, Heine-Borel theorem. Continuity and differentiability: continuity and its characterizations, Bolzano's intermediate value theorem, differentiability on the real line, characterisations, Caratheodory's theorem, Rolle's theorem, mean value theorem, differentiability in higher dimensions, partial derivatives, characterizations.	9	3	

4	**Real Analysis III.** Convex sets, subgraph and epigraph of a function, concavity and convexity, characterizations, contour sets, quasiconcavity and quasiconvexity.	6	1	
5	**Optimisation.** Optimisation problems, existence of solutions, Weierstrass theorem. Motivating examples: utility maximisation, cost minimisation, profit maximisation, exchange economy.	4		
6	**Unconstrained optimisation.** Unconstrained optimisation problems, unconstrained maximum, local maximum, unconstrained local maximum, strict local maximum, first-order condition, second-order condition.	4	1	
7	**Optimisation with equality constraints.** Optimisation problems with equality constraints, theorem of Lagrange, constraint qualification under equality constraints, cookbook procedure and its limitations, sufficient conditions for cookbook procedure. Applications: utility maximisation with Cobb-Douglas utility function, cost minimization.	6	2	
8	**Optimisation with inequality constraints.** Optimisation problems with equality constraints, theorem of Kuhn and Tucker, constraint qualification under inequality constraints, cookbook procedure and its limitations, sufficient conditions for cookbook procedure. Applications: utility maximisation with perfect substitutes utility function, cost minimization.	6	2	
	Total	48	12	

Pedagogical approach:

Classroom teaching and problem-solving sessions.

Evaluation criteria:

Minor 1: Written Examination - 30% [Syllabus: 1-2, Learning outcomes: 1-3]
Minor 2: Written Examination - 30% [Syllabus: 3-4, Learning outcomes: 1-3]
Major: Written Examination - 40% [Syllabus: 5-8, Learning outcomes: 1-4]

Learning outcomes:

1. To understand the core concepts of mathematics.
2. To develop mathematical sophistication.
3. To write proofs rigorously.
4. To apply concepts and results in economic applications.

Core readings:

1. S. Abbott. “Understanding Analysis” (2016) (SA)
2. R. Sundaram. “A First Course in Optimization Theory” (1996) (RS)

Additional readings:

1. R. Bartle, D. Sherbert. “Introduction to Real Analysis” (2010) (RB)
2. C. Simon, L. Blume. “Mathematics for Economists” (2018)

Module-wise chapters:

1. Module 1: SA, Chapter 1 [Learning outcomes: 1]
2. Module 2: SA, Chapters 1 and 2; RS: Chapter 1 [Learning outcomes: 1-3]
3. Module 3: SA, Chapters 3-5; RS: Chapter 1 [Learning outcomes: 1-3]
4. Module 4: RS, Chapters 7-8 [Learning outcomes 1-3]
5. Module 5: RS, Chapters 2 and 3 [Learning outcomes: 1-3]
6. Module 6: RS, Chapter 4 [Learning outcomes: 1-3]
7. Module 7: RS, Chapters 4 and 5 [Learning outcomes: 1-4]
8. Module 8: RS, Chapter 6 [Learning outcomes: 1-4]

Course prepared by: Sanyyam Khurana

Student responsibilities: Attendance, feedback, discipline: as per university rules.

Course reviewers:

1. Abhijit Banerji. Professor, Department of Humanities and Social Sciences, Indian Institute of Technology Delhi
2. Sudhir Shah. Professor, Department of Economics, Delhi School of Economics