Course title: Linear Algebra and Dynamic Optimisation			
Course code: MPE 114	No. of credits: 4	L-T-P: 51-9-0	Learning hours: 60
Pre-requisite course code and title: MPE 117; Real Analysis and Optimization			
Department: Department of Policy and Management Studies			
Course coordinator: Dr Malvika Tyagi	**Course instructor: Dr Malvika Tyagi**		
Contact details: malvika.tyagi@terisas.ac.in			
Course type: Core	**Course offered in:** Semester 2		
Course description: This course develops fundamentals of certain areas of mathematics that are relevant to growth economics, game theory, resource economics, and econometrics. The course covers vector spaces, linear transformations, inner product spaces, spectral theory, difference and differential equations, dynamicoptimisation in discrete and continuous time.			
Course objective: 1. To develop the core concepts of mathematics that are used in economics. 2. To develop mathematical sophistication and proof writing. 3. To build analytical and technical skills by rigorous analysis. 4. To apply these techniques and skills in economic applications.			

Course contents

S.No	Topics	L	T	P
1	**Linear Algebra I.** Vector spaces: Real vector space, subspace, span, linear independence and dependence, characterizations, sum, direct sum. Bases and dimension: basis, standard basis, finite dimensional vector space, existence of a basis and its characterization, dimension and its properties.	6	1	
2	**Linear Algebra II.** Linear transformation: linear transformation and its types, transformation matrix, change of basis. Null space and range: null space, range, nullity, rank, fundamental theorem of linear algebra. Inner product spaces: inner product space, normed space, orthogonality, Pythagorean theorem, Cauchy-Schwarz inequality, Parallelogram equality, polarisation identity, othronormal bases, Gram-Schmidt. Spectral theory: eigenvalue, eigenvector, spectrum, diagonalization.	8	1	
3	**Discrete dynamics.** Linear systems in one dimension: difference equations, characterisation, steady state equilibria, existence results, stability of steady states, local stability, global stability. Linear systems in multi dimensions: characterisation, steady states, existence of equilibria, stability analysis.	4		
4	**Differential equations.** First-order ordinary differential equations, solutions, homogeneous equations, stability, saddle points, Picard's existence theorem.	7	1	

5	**Preliminaries for dynamic optimisation** Preliminaries: metric spaces, correspondences, upper hemicontinuity, lower hemicontinuity, contraction mapping theorem, Blackwell's sufficient conditions, theorem of the maximum.	8		
6	**Dynamic optimisation in discrete time.** Finite horizon: Dynamic programming, value function, existence result, Bellman equation. Infinite horizon: Dynamic programming (sequence problem), value function (recursive formulation), Bellman equation (principle of optimality), Euler equation, transversality condition, existence result, concavity of value function. Application: consumption-saving problem, optimal growth model, cake eating problem, tree cutting problem.	10	3	
7	**Optimal control theory.** Dynamic optimisation in continuous time, Hamiltonian function, maximum principle, Mangasarian's sufficient conditions, Arrow's sufficient conditions, transversality conditions. Application: optimal growth model in continuous time.	8	3	
	Total	51	9	

Pedagogical approach:

Classroom teaching and problem-solving sessions.

Evaluation criteria:

Minor 1: Written Examination - 30% [Syllabus: 1-2, Learning outcomes: 1-3]
Minor 2: Written Examination - 30% [Syllabus: 3-4, Learning outcomes: 1-4]
Major: Written Examination - 40% [Syllabus: 5-7, Learning outcomes: 1-4]

Learning outcomes:

1. To understand the core concepts of mathematics.
2. To develop mathematical sophistication.
3. To write proofs rigorously.
4. To apply concepts and results in economic applications.

Core readings:

1. S. Axler. "Linear Algebra Done Right" (1997) (SA)
2. O. Galor. "Discrete Dynamical Systems" (2003) (OG)
3. G. Simmons, S. Krantz. "Differential Equations: Theory, Technique, and Practice" (2007) (SK)
4. D. Acemoglu. "Introduction to Modern Economic Growth" (DA)
5. R. Sundaram. "A First Course in Optimization Theory" (1996) (RS)

Additional readings:

1. A. Chiang. "Elements of Dynamic Optimization"
2. G. Strang. "Linear Algebra and its Applications", 3rd edition (1998)
3. N. Stokey, R. Lucas. "Recursive Methods in Economic Dynamics" (1988)

Module-wise chapters:

1. Module 1: SA, Chapters 1-3 [Learning outcomes: 1-3]
2. Module 2: SA, Chapters 3 and 10 [Learning outcomes: 1-3]
3. Module 3: OG, Chapters 1-2 [Learning outcomes: 1-3]
4. Module 4: SK, Chapters 3, 10-11 [Learning outcomes: 1-4]

5. Module 5: DA, Chapter 6 [Learning outcomes: 1-3]
6. Module 6: DA, Chapter 6; RS, Chapters 11-12 [Learning outcomes: 1-4]
7. Module 7: DA, Chapter 7 [Learning outcomes: 1-4]

Course prepared by: Sanyyam Khurana

Student responsibilities: Attendance, feedback, discipline: as per university rules.

Course reviewers:

1. Abhijit Banerji. Professor, Department of Humanities and Social Sciences, Indian Institute of Technology Delhi
2. Sudhir Shah. Professor, Department of Economics, Delhi School of Economics